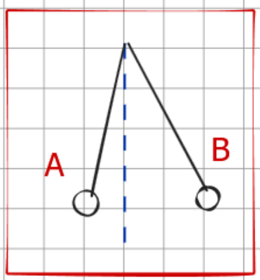
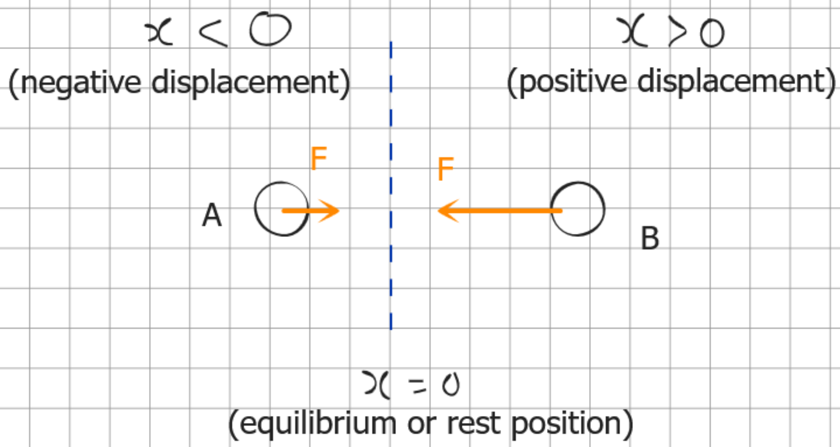
SHM (Simple Harmonic Motion) Basics

10th July

Definition: Motion for which the ACCELERATION of the object is DIRECTLY PROPORTIONAL to the DISPLACEMENT, and is always in the OPPOSITE DIRECTION to displacement.

Alternative wording: Motion for which the acceleration of the object is directly proportional to the displacement, and always acts towards the equilibrium (rest) position.



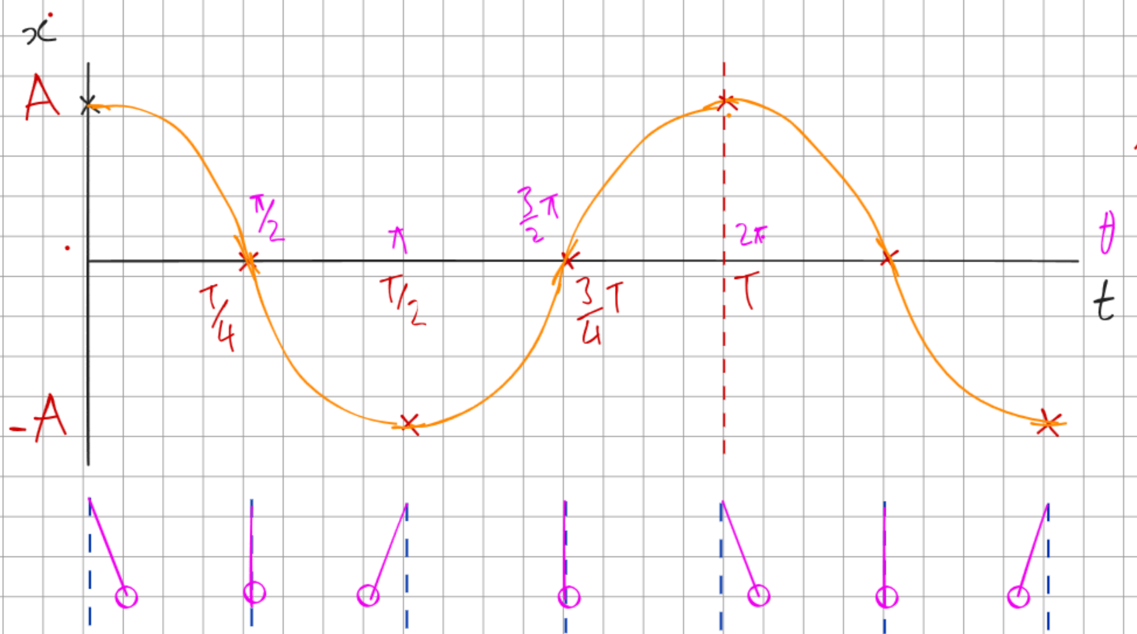


- Note that if acceleration is proportional to displacement and in the opposite direction, then so is the resultant force causing the motion (according to N2L)

In SHM:

$$a \propto -x$$

$$F \propto -x$$



The graph above is a displacement time graph for an object undergoing SHM.

It has the equation: $x = A \cos\theta$ (where A is AMPLITUDE)

We know that: $\omega = \frac{\theta}{t}$ $\theta = \omega t$

Hence $\boxed{x = A \cos \omega t}$ (where t is time elapsed)

given in data sheet

Note this is often used in conjunction with: $\omega = 2\pi f$, $f = 1/T$

$$v = -\omega A \sin \omega t$$

Magnitude of maximum velocity: $\boxed{v_{max} = \omega A}$ given

$$a = -\omega^2 A \cos \omega t$$

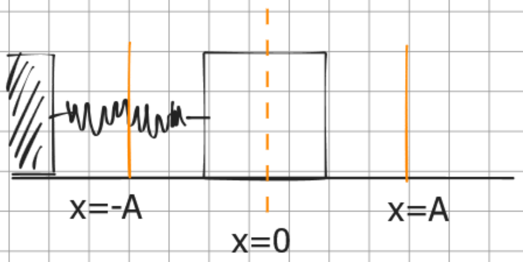
$\boxed{a = -\omega^2 x}$ given

Magnitude of maximum acceleration: $\boxed{a_{max} = \omega^2 A}$ given

Mass on a Spring

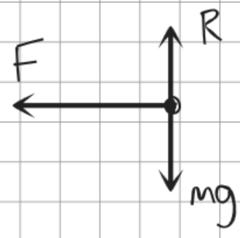
8th Sep

Useful equations: $F = k\Delta L$, $F = ma$, $a = -\omega^2 x$, $\omega = 2\pi f = \frac{2\pi}{T}$



Mass oscillating horizontally on a frictionless plane.

When the mass has its maximum positive displacement:



Assuming that the spring obey's Hooke's law:

$$F = -k\Delta L$$

$$F = -kx$$

(x is displacement, which is the same as change in length here. The minus shows the force is acting to the left in this instance).

From N2L

$$ma = -kx$$

$$a = -\frac{k}{m}x$$

We also know that in SHM:

$$a = -\omega^2 x$$

$$\cancel{-}\omega^2\cancel{x} = \cancel{-}\frac{k}{m}\cancel{x}$$

$$\omega^2 = k/m$$

$\omega = \frac{2\pi}{T}$ hence:

$$\left(\frac{2\pi}{T}\right)^2 = k/m$$

period of oscillation

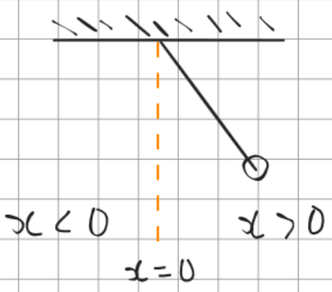
$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

oscillating mass

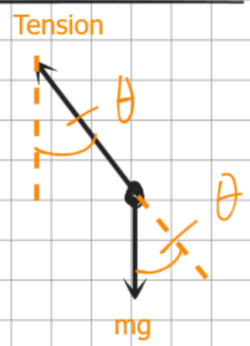
force per unit displacement (force required to displace the system by 1 m) aka the 'spring constant'

Period of a Pendulum

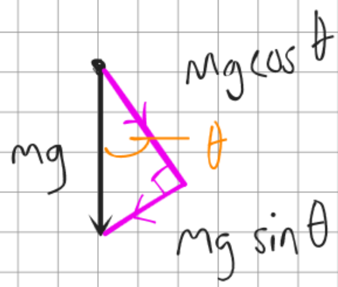
15th Sep



Free Body Diagram



Resolving the weight into two components; parallel to string and normal to string:



The restoring force acts tangentially to the arc that the pendulum moves through.

We are going to assume a small angular displacement, which means the chord length is the same as the arc length.

We know, in radians; $\theta = \frac{s}{l}$ so $s = l\theta$.

Our restoring force:

$$F = -kx$$

$$F = -ks$$

$$F = -kl\theta$$

We also know that: $F = -mg\sin\theta$

Hence; $-kl\theta = -mg\sin\theta$

At small angles: $\sin\theta \approx \theta$

Giving

$$\cancel{-}kl\cancel{\theta} = \cancel{-}mg\cancel{\theta}$$

$$\frac{m}{k} = \frac{l}{g}$$

We know that for an oscillating mass on a spring: $T = 2\pi\sqrt{m/k}$

Which then becomes: $\boxed{T = 2\pi\sqrt{l/g}}$

length of pendulum measured from attachment point to CoM of mass on end of string

gravitational field strength

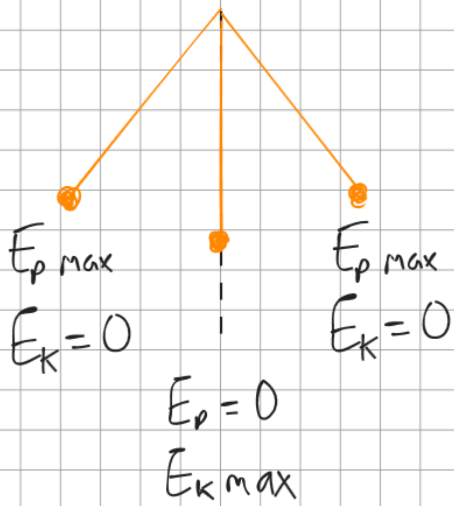
period of oscillation

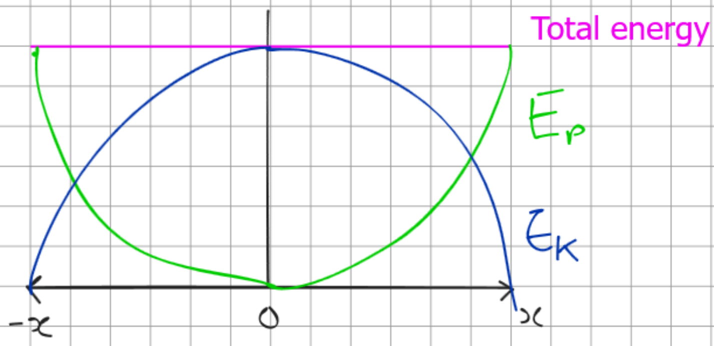
Energy in SHM

29th Sep

In a FREE OSCILLATION (one with no resistive forces) the total energy of the oscillating system remains constant.

Total energy = potential energy + kinetic energy





The work done to displace the pendulum to its amplitude tells us the total energy stored in the system.

$$W = \int_0^A F\,dx \qquad \text{where} \qquad F = kx$$

$$= \int_0^A kx\,dx = \left[\tfrac{1}{2}kx^2\right]_0^A$$

$$= \tfrac{1}{2}kA^2 - 0$$

Total energy $= \frac{1}{2}kA^2$

Therefore: $\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$ $\quad \left(\omega^2 = k/m\right)$

$$\boxed{v = \pm\omega\sqrt{A^2 - x^2}}$$

This equation tells us the velocity of a freely oscillating object at any displacement.

Free and Forced Oscillations

2nd Oct

Free oscillations have NO RESISTIVE FORCES and DISSIPATE no energy to their surroundings.

Free oscillations have a constant amplitude.

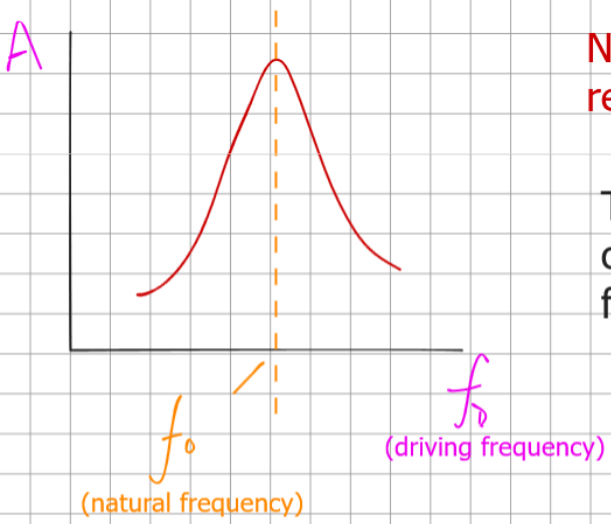
A system oscillating freely will do so at its NATURAL FREQUENCY.

This natural frequency depends on the physical properties of the sustem i.e. pendulum length, spring constant, mass etc

A FORCED OSCILLATION is driven by a PERIODIC DRIVING FORCE (an external force with its own frequency, the driving frequency).

If the frequency of the driving force matches the natural frequency of the oscillating system we get RESONANCE.

At resonance we get a sharp and sudden increase in AMPLITUDE. This is due to energy being put into the system.



Note: you may see the natural frequency referred to as the 'resonant frequency'.

This graph shows how the amplitude of oscillations varies when we change the frequency of the driving force.

We get resonance when:

$$f_D = f_0$$

Damped Oscillations

6th Oct

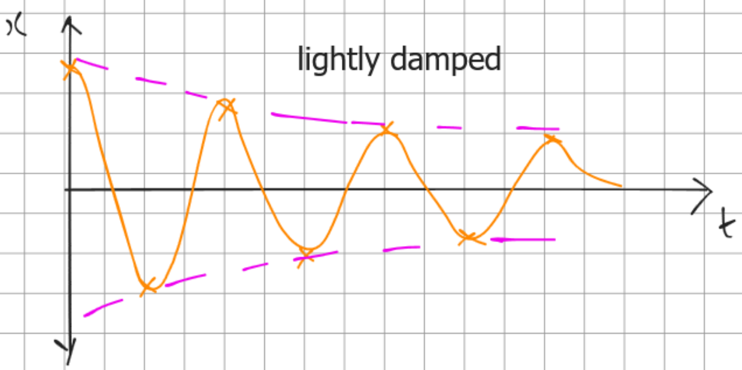
For most oscillations in the real world there are RESISTIVE FORCES.

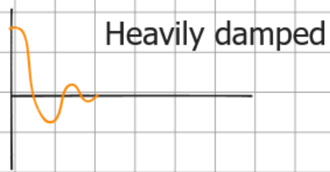
These resistive forces (friction, drag etc) remove energy from the oscillating system. This energy is usually dissipated to the surroundings (and increases the thermal store of the surroundings).

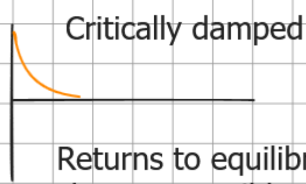
$$\text{Total energy} \propto (\text{Amplitude})^2$$

So, as energy is dissipated, the amplitude decreases over time.

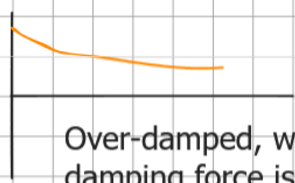
This process of decreasing the amplitude is called DAMPING.



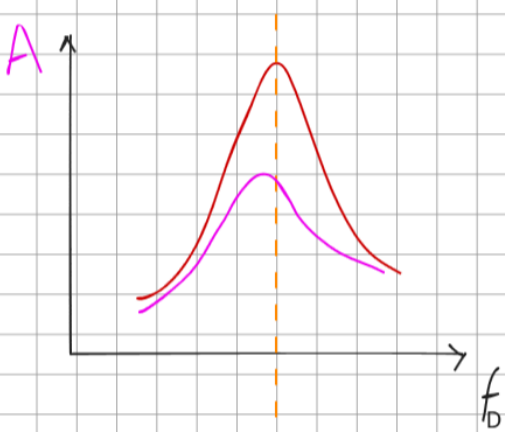




Returns to equilibrium in the shortest possible time



Over-damped, where the damping force is similar in magnitude to the restoring force



- Damping lowers the amplitude at ALL frequencies, as energy is dissipated from the system
- Damping reduces the maximum amplitude
- Can slightly reduce the natural frequency
- Reduces the sharpness of resonance, giving a broader peak