Gravitational Fields Basics

20th Oct

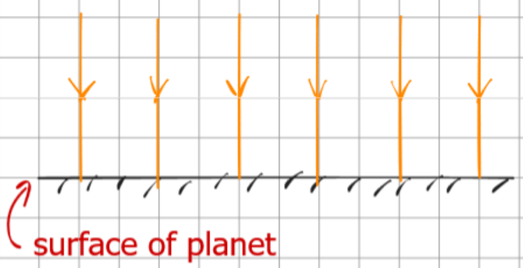
Force field: a region in space when an object experiences a non-contact force

Gravitational field: region where a massive object experiences a gravitational force

Gravitational forces are: always attractive
only significantly affects large masses

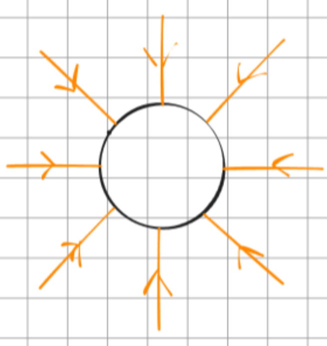
- Fields are vectors and are represented by FIELD LINES with ARROWS.
- Field lines are CONTINUOUS (no gaps or breaks)
- The density of field lines in an area represents the STRENGTH of the field
- In gravitational fields the direction is determined by the direction of the force a mass would experience.

Uniform Field



- Close to the surface of a large spherical object we assume that the field is UNIFORM (has the same strength everywhere).
- Field lines are PARALLEL and EQUIDISTANT (so the field line density is constant).

Radial Fields



- Radial field lines would meet at the centre of mass of a spherical object.
- We treats fields around planets/moons as radial UNLESS we are very close to the surface
- Field is strongest closest to the object (field line density is greatest/lines are closest together)

Gravitational field strength: gravitational force per unit mass

$$g = \frac{F}{m}$$ in $N\,kg^{-1}$ $\boxed{\text{vector quantity}}$

It is a property of the space around a massive object.

Gravitational potential: gravitational potential energy per unit mass

$$V = \frac{E_p}{m} \quad \text{in J kg}^{-1}$$

$\boxed{\text{scalar quantity}}$

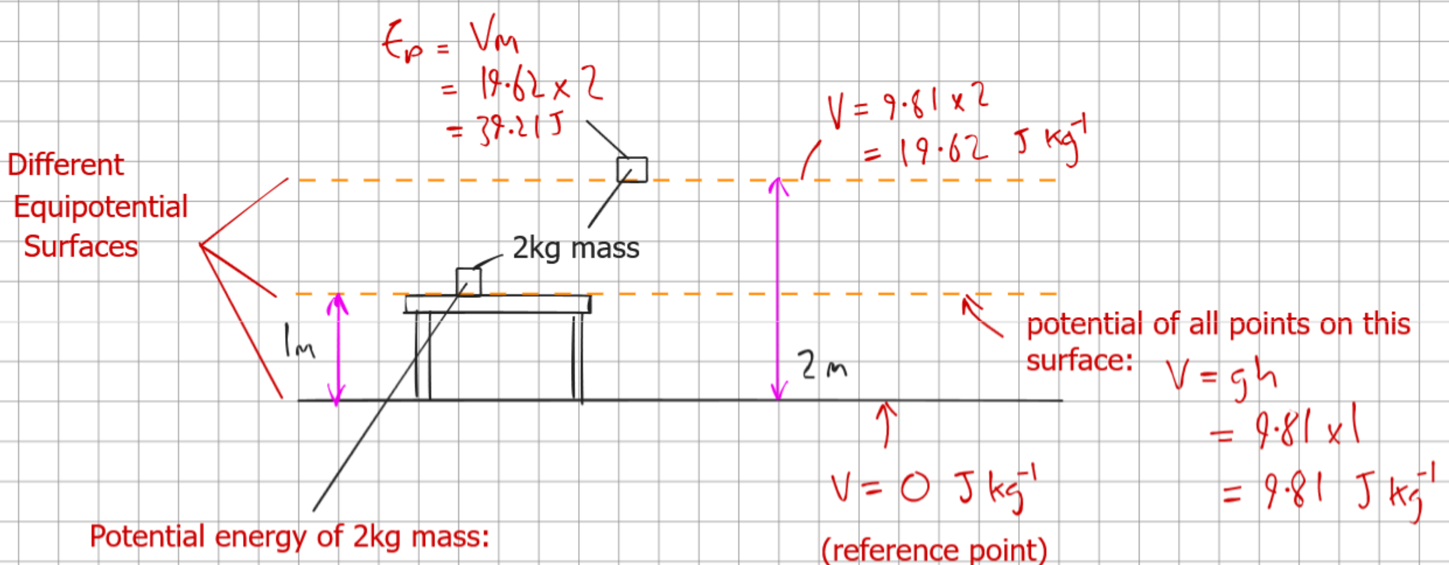
It is a property of the space around a massive object.

Gravitational Potential Energy: the work done to move an object from a reference point to its current position in a gravitational field.

Note: in uniform fields the reference point (where GPE is 0 J) is taken as the ground.

In a uniform field: $\Delta E_p = mg\Delta h$

and so by definition: $\Delta V = g\Delta h$ $\left(\text{as } \Delta V = \frac{\Delta E_p}{m}\right)$



Potential energy of 2kg mass:

$$E_p = Vm$$
$$= 9.81 \times 2$$
$$= 19.62\ J$$

All points on an equipotential line/surface have the SAME GRAVITATIONAL POTENTIAL.

No work done against gravity to move an object from one point to another along an equipotential.

In uniform fields equipotentials are flat surfaces, but in radial fields they are spherical surfaces.

Equipotentials are always perpendicular to field lines.

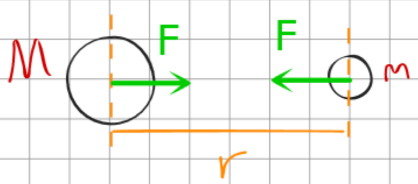
Newton's Law of Gravitation

23rd Oct

The force of gravity between two point objects is:

- proportional to the product of the masses of the objects
- inversely proportional to the square of the separation of the objects

Note: when dealing with large objects we use the separation of their centres of mass



$$F \propto Mm \qquad F \propto \frac{1}{r^2}$$

$$F \propto \frac{Mm}{r^2}$$

$$F = \frac{GMm}{r^2}$$

magnitude of the gravitational force

distance between centres of mass of the objects

G is the universal gravitational constant

$6.67 \times 10^{-11}\ N\,m^2\,kg^{-2}$

The gravitational field strength at a distance r from the CoM of an object with mass M is:

$$g = \frac{F}{m} \quad \text{(by definition)}$$

$$\boxed{g = \frac{GM}{r^2}}$$

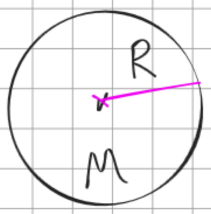
So when an object of mass m is placed at a location with this field strength, it experiences a force:

$$F = mg = \frac{GMm}{r^2}$$

Field Strength and Density

10th Nov

In a radial field; $g = \frac{GM}{r^2}$ where r is the distance from the centre of mass



At the surface of this planet: $g = \frac{GM}{R^2}$

We know: $\rho = M/V$ so $M = \rho V$

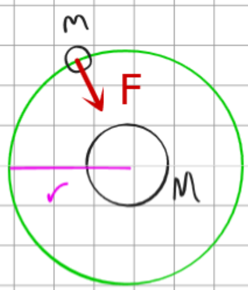
Where, assuming a spherical planet, $V = \frac{4}{3}\pi R^3$

So: $M = \frac{4}{3}\rho\pi R^3$ and $\boxed{g = \frac{4}{3}G\rho\pi R}$

Orbits

13th Nov

When an object is in a circular orbit, gravity provides the required centripetal force.



Resultant force on m: $F = \frac{GMm}{r^2}$

F is the centripetal force here so: $F = \frac{mv^2}{r}$

Hence: $$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

v in this expression tells us the required linear speed for an object at a distance 'r' from a large mass 'M' to stay in a STABLE ORBIT.

We can see that $v \propto \frac{1}{\sqrt{r}}$ meaning that the velocity required for a stable orbit decreases as orbital radius increases.

At greater distances from an object the gravititational field strength gets weaker.

This means that the gravitational field can provide a lower centripetal force. Hence, to stay moving in a circle you would have to slow down as you move further away.

Orbital radius: measured between the orbiting object and the centre of mass of the planet/moon/star

Orbital height: measured between the orbiting object and the surface of the planet/moon/star

Kepler's Third Law: (For any objects orbiting the same body) the square of the orbital time period is proportional to the cube of the orbital radius.

$$T^2 \propto r^3$$

$$\frac{T^2}{r^3} = \text{constant}$$

the value of the constant depends on the object that is being orbited

It can be shown that:

$$F = \frac{GMm}{r^2}$$

$$\cancel{m} r\omega^2 = \frac{GM\cancel{m}}{r^2}$$

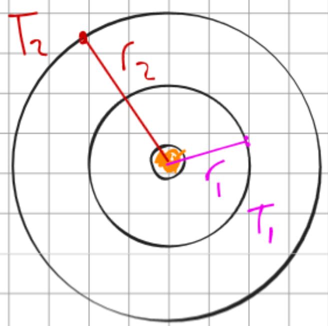
$$\omega^2 r^3 = GM$$

$$\left(\frac{2\pi}{T}\right)^2 r^3 = GM$$

$$\frac{4\pi^2}{T^2} r^3 = GM$$

$$T^2 = \frac{4\pi^2}{GM} \times r^3$$

Learn how to do this!



$$\frac{T_1^2}{r_1^3} = \text{constant}$$

$$\frac{T_2^2}{r_2^3} = \text{constant}$$

$$\boxed{\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}}$$

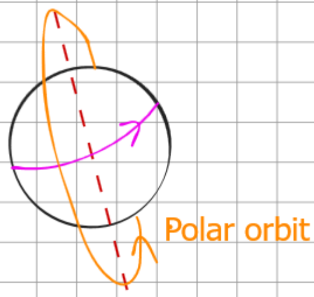
Note: 'M' represents the mass of the object being orbited
When using any equation with 'G', all other quantities must be in standard units

A GEOSTATIONARY (or geosynchronous) orbit has the following features:

- An orbital time period of 24 hours
- Orbit above the equator (in equatorial plane)

This means the satellite stays above the same location on the surface of the Earth. Both the satellite and the Earth have the same ANGULAR VELOCITY.

Satellites in these orbits are typically used for communication.



Another common type of orbit is a POLAR ORBIT.

This orbit is typically lower and faster than a geostationary one.

It can cover the whole surface of the Earth as it rotates.

Typically used for mapping, surveillance and monitoring weather.

Grav Potential in Radial Fields

20th Nov

If we ensure at all times that the force exerted on the object to move it (F) from (r_1) to (r_2) is equal to the magnitude of the force of gravitational attraction (F_g), then all of the work done changes the gravitational potential energy of the object.

If $|F| = |F_g|$ then $W = \Delta E_p$

From Newton's Law of Gravitation: $F = \dfrac{GMm}{r^2}$

$$W = \Delta E_p = \int_{r_1}^{r_2} F\,dr$$

$$= \int_{r_1}^{r_2} \frac{GMm}{r^2}\,dr$$

$$\Delta E_{p\ r_1 \to r_2} = \frac{GMm}{r_1} - \frac{GMm}{r_2}$$

Let us consider moving an object so far away from the CoM of the planet that it escapes the gravitational field.

$$\Delta E_{p\,(r_1 \to \infty)} = \frac{GMm}{r_1} - \frac{GMm}{\infty}$$

$$= \frac{GMm}{r_1}$$

$$\Delta E_p = \frac{GMm}{r}$$

This tells us the change in GPE to move an object from 'r' to outside of the field

GPE of an object just outside of the field $= E_p + \Delta E_p$

GPE at its original position

change in GPE when moved to just outside of the field

We define the potential energy at an 'infinite' distance away as 0.

$$0 = E_p + \Delta E_p$$

$$E_p = -\Delta E_p$$

$$\boxed{E_p = -\frac{GMm}{r}.}$$

This expression lets us calculate the gravitational potential energy of an object mass 'm', placed a distance 'r' from the CoM of an object mass 'M'.

By definition: $V = \frac{E_p}{m}$

Hence, the gravitional potential of a point a distance 'r' away from an object with mass 'M' is:

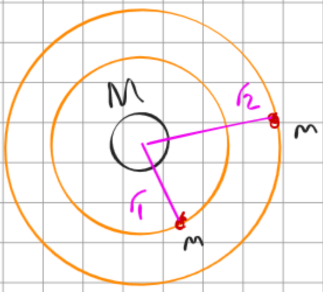
$$\boxed{V = -\frac{GM}{r}}$$

Gravitational potential: work done when moving a unit mass from infinite distance to a point in a gravitational field

Values of gravitational potential are negative because their maximum value is 0 at an infinite distance away.

Energy Changes in Orbits

27th Nov



Here we have an object mass m orbiting a planet mass M at two different orbital heights, r_1 and r_2 .

For any transition between orbits there will be a CHANGE IN POTENTIAL ENERGY and a CHANGE IN KINETIC ENERGY.

The sum of these changes tells us the total work that needs to be done to the change in orbit.

$$W = \Delta E_p + \Delta E_k$$

If the work required is a positive value, then we need to do work. This is done by burning fuel.

Kinetic Energy in Orbits

In orbit the gravitational force acts as the centripetal force:

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

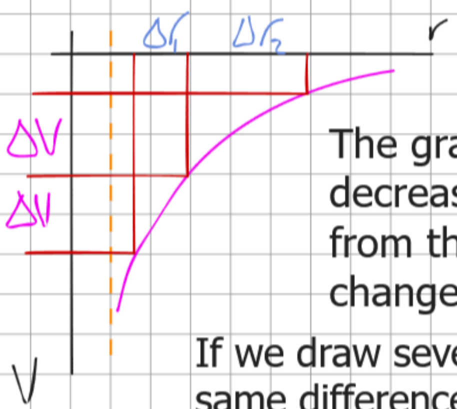
$$mv^2 = \frac{GMm}{r}$$

So Kinetic energy;

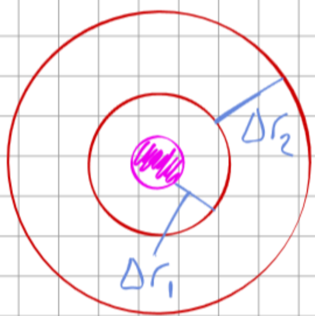
$$E_k = \frac{1}{2} mv^2$$

$$E_k = \frac{1}{2} \frac{GMm}{r}$$

Field Strength and Potential



$$V = -\frac{GM}{r}$$



The gravitational field strength decreases as we move further away from the planet. This means to have the same change in potential, we would have to move a greater distance.

If we draw several equipotentials that are separated by the same difference in potential, they will eventually be spaced further and further apart.

The rate at which potential changes as distance changes tells us the GRAVITATIONAL FIELD STRENGTH. Hence, the gradient of the graph of V against r give us the gravitational field strength.

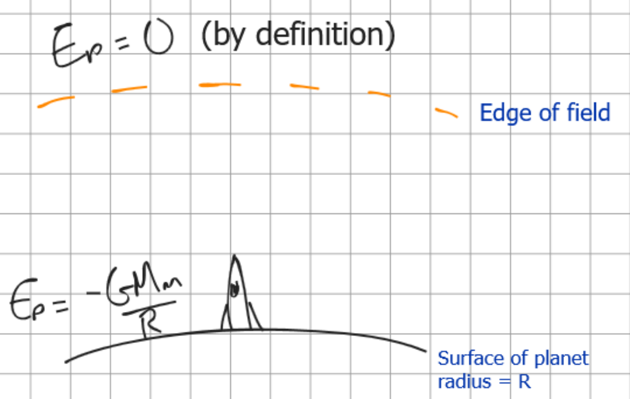
Escape Velocity

27th Nov

If an object is launched from a surface then its kinetic energy decreases as its gravitational potential energy increases.

If we assume that no energy is dissipated we can state that the loss in kinetic energy is equal to the gain in potential energy (similar to throwing a ball upwards).

Potential energy of an object = $E_p = Vm = -\frac{GMm}{r}$ $\left(\text{as } V = -\frac{GM}{r}\right)$



Change in potential energy required:

$$\Delta E_p = 0 - \left(-\frac{GMm}{R}\right)$$

$$\Delta E_p = \frac{GMm}{R}$$

Change in kinetic energy:

$$\Delta E_k = 0 - \frac{1}{2}mv^2$$

$$\Delta E_k = -\frac{1}{2}mv^2$$

Total change in energy = $\Delta E_p + \Delta E_k = 0$

$$0 = \frac{GMm}{R} - \frac{1}{2}mv^2$$

Kinetic energy at launch: $\frac{1}{2}mv^2 = \frac{GMm}{R}$:Potential energy we need to gain

Escape velocity: $v = \sqrt{\frac{2GM}{R}}$

This specific value of v tells us the launch speed required for an object to gain enough potential energy to escape the gravitational field.

We call this ESCAPE VELOCITY.