On 'Ideal' Gases

Real gases DO NOT behave like an ideal gas.
Their behaviour is however sufficiently close to explain common patterns. An ideal gas presents a SIMPLIFIED MODEL based on a series of ASSUMPTIONS:

- An ideal gas consists of a large number of IDENTICAL, SMALL, HARD SPHERICAL molecules.
- The volume of each molecule is NEGLIGIBLE compared to the volume of the container.
- All collisions between the molecules themselves and the container are ELASTIC and FRICTIONLESS (no energy loss)
- The movement of the molecules adhere to NEWTONIAN MECHANICS (molecules obey Newtons laws)
- The MEAN distance between molecules is MUCH LARGER than the size of an individual molecule.
- The molecules move in RANDOM DIRECTIONS with a DISTRIBUTION OF VELOCITIES around a MEAN VELOCITY.
- Molecules move in STRAIGHT LINES between collisions.
- The time taken for a collision to occur is NEGLIGIBLE compared to the time between collisions.
- There are NO INTERMOLECULAR FORCES between gas molecules outside of collisions.

The most important property of an ideal gas is the notion of random motion.

This was Robert Brown whilst he was observing pollen grains suspended on a Petri dish of water. Einstein later explained this motion using Atomic Theory.

Volume

Random motion means that there is NO PREFERRED DIRECTION, so molecules spread out to fill the container.
If the dimensions of the container change, the gas continues to fill the whole volume and its MOTION will CHANGE ACCORDINGLY.

Temperature

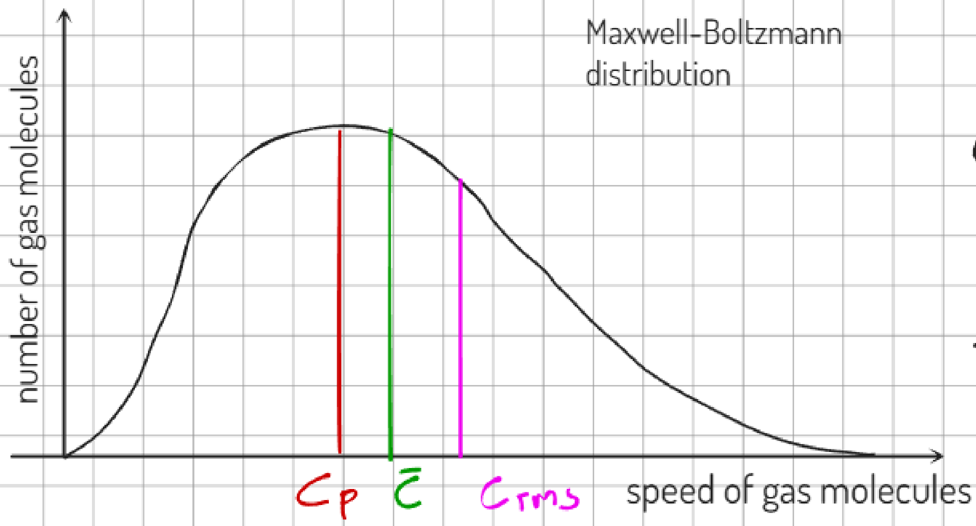
There are NO INTERMOLECULAR FORCES, so increasing temperature will affect ONLY THE KINETIC ENERGY of the molecules.
This will lead to an INCREASE in their AVERAGE SPEED.

Pressure

Macroscopic pressure is defined as: The force applied over a given area.

This force is due to collisions between the MOLECULES and the WALLS of the CONTAINER.
The changing momenta of the rebounding molecules (elastically colliding) every second causes a RESULTANT FORCE across the INNER SURFACE AREA of the container in ALL DIRECTIONS
(due to the random motion)

If motion were not random then some surfaces may experience more pressure than others.



6 particles

600 ms^{-1}, 650 ms^{-1},
650 ms^{-1}, 700 ms^{-1},
725 ms^{-1}, 750 ms^{-1}

The average velocity is ZERO at any temperature.

The MOST PROBABLE SPEED (c_p) is 650 m/s

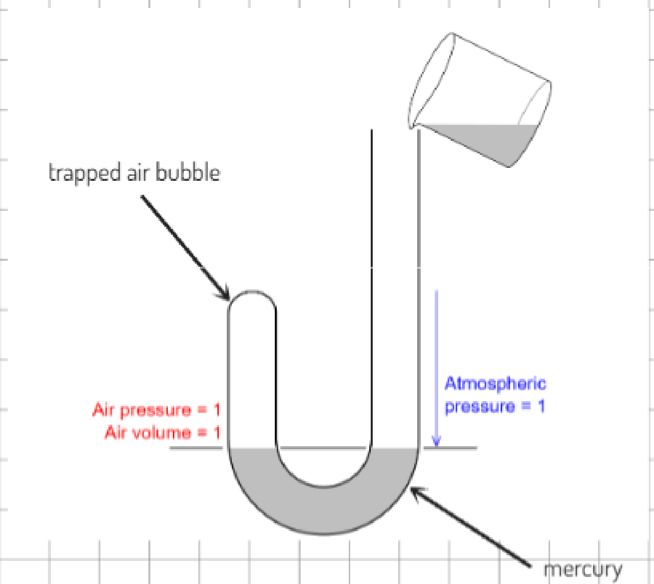
The MEAN SPEED ($\bar{c}$) is 679 m/s

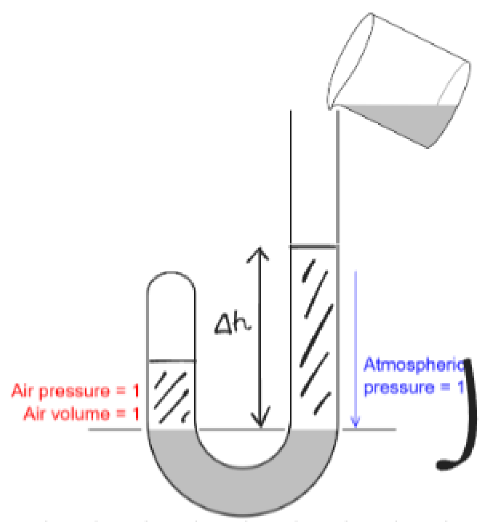
The ROOT MEAN SQUARE SPEED (c_{rms}) is 681 m/s

$$c_{rms} = \left(\frac{(600^2 + 650^2 + 650^2 + 700^2 + 725^2 + 750^2)}{6}\right)^{1/2}$$

Gas Laws

Boyle's Law





$p = \frac{F}{A}$

$p = \frac{mg}{A}$

$p = \frac{\rho V g}{A}$

$\boxed{p = \rho h g}$

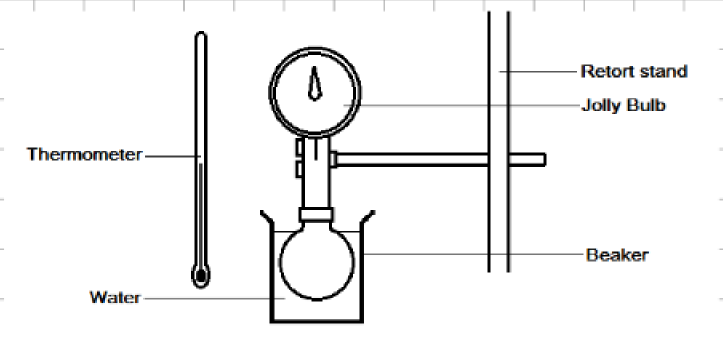
pressure of air bubble	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
volume of air bubble	1.0	0.909	0.833	0.769	0.714	0.667	0.625	0.588	0.556

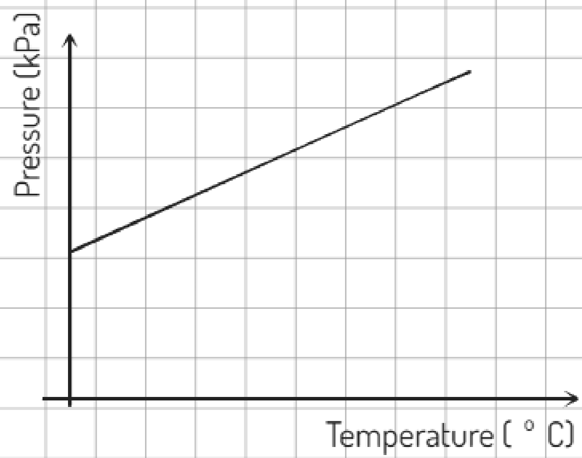
$p \propto \frac{1}{V}$ $\quad p = \frac{\text{constant}}{V}$ $\quad pV = \boxed{\text{constant}}$

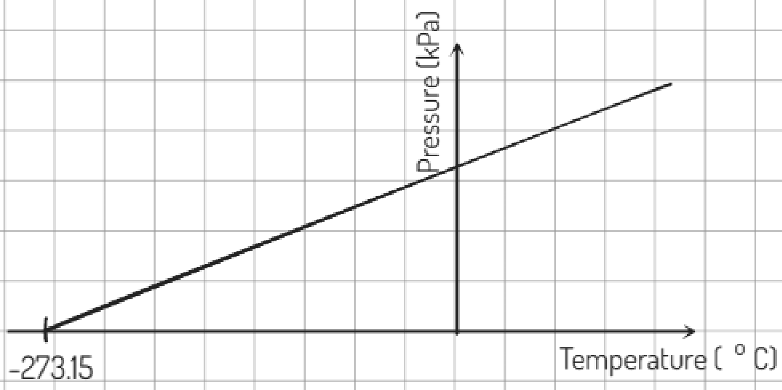
to prove inverse proportionality, select three pairs of points and multiply them to produce the same constant

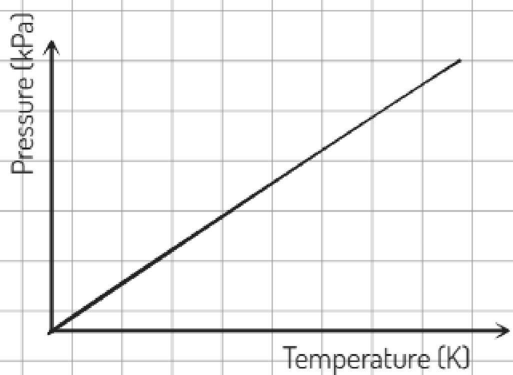
NOTE: Boyle's law only applies to a gas kept at CONSTANT TEMPERATURE

Amonton's Law









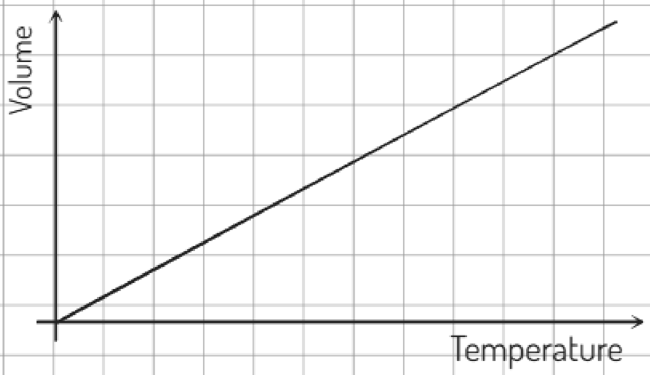
$p \propto T$ $\quad p = \text{constant} \times T$ $\quad \frac{p}{T} = \text{constant}$

to prove direct proportionality, select three pairs of points and divide one by the other to produce the same constant

NOTE: Amonton's law only applies to a gas kept at CONSTANT VOLUME

Charles' Law

Observation of a balloon in hot and cold environments.



$V \propto T$ $\quad\quad V = \text{constant} \times T$

$$\frac{V}{T} = \text{constant}$$

NOTE: Charles' law only applies to a gas kept at CONSTANT PRESSURE

Combined Gas Laws

$\frac{p}{T} = \text{constant}$ $\quad\quad \frac{V}{T} = \text{constant}$

$pV = \text{constant}$

$$\boxed{\frac{pV}{T} = \text{constant}}$$

$$\boxed{\frac{p_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}}$$

Gas Laws

Avogadro's Law

Under conditions where temperature and pressure remain constant, it can be assumed that the volume of a gas is proportional to the number of moles of gas present in a sample.

$V \propto n$ ← number of moles

$$\frac{V_1}{n_1} \propto \frac{V_2}{n_2}$$

$$\frac{p_1 V_1}{n_1 T_1} = \text{constant}$$

$$pV = nRT$$

$N m^{-2} m^{3} = mol\, K\, R$

$N m\, mol^{-1} K^{-1} = R$

where $\boxed{R}$ is $\boxed{8.31\ J\,mol^{-1}K^{-1}}$

(the molar gas constant)

$\boxed{N_A}$ – number of particles in one mole of substance

$\boxed{6.02 \times 10^{23}\ mol^{-1}}$

$$\boxed{k} = R/N_A = \frac{8.31}{6.02 \times 10^{23}} = \boxed{1.38 \times 10^{-23}\ JK^{-1}}$$

(Boltzmann's constant)

$$R = k N_A$$

$$pV = k \boxed{N_A n} T$$

the number of particles

$$pV = \boxed{N} kT$$

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Molar/molecular mass

$$\boxed{M_m} = N_A \boxed{m}$$

molar mass — molecular mass

$\boxed{M_g}$ mass of gas

$$n = \frac{M_g}{M_m}$$

$$N = \frac{M_g}{m}$$

$$pV = nRT$$

becomes:

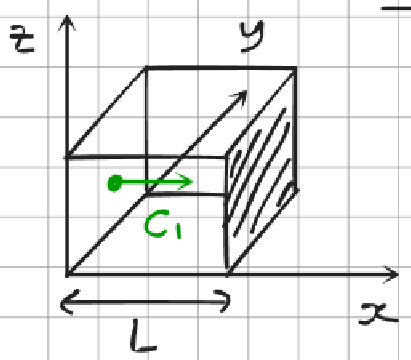
$$pV = \frac{M_g RT}{M_m}$$

$$pV = NkT$$

becomes

$$pV = \frac{M_g kT}{m}$$

Kinetic Theory of Gases



A particle, moving along the x axis with velocity c_1

- hard, spherical molecule
- volume of particle is negligible compared to container

In the box are N particles with velocities of $c_1, c_2, c_3 \cdots\cdots c_N$

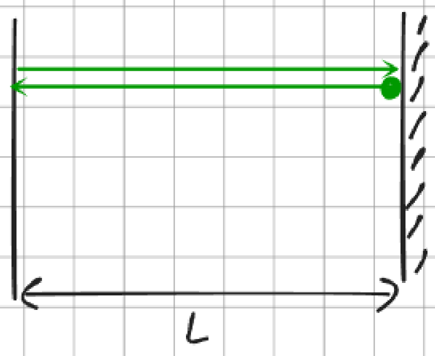
- elastic collision
- Newtons laws apply

Assuming elastic collisions

$$\Delta p = (-mc_1) - mc_1$$

$$\boxed{\Delta p = -2mc_1} \quad (1)$$

Assuming negligible volume of molecules



distance between collisions with the surface = $2L$

$$t = \frac{d}{c_1} = \frac{2L}{c_1}$$

$$\boxed{\frac{1}{t} = \frac{c_1}{2L}} \quad (2)$$

RATE OF COLLISION

Assuming Newton's Laws apply

$$F = \frac{\Delta p}{t} = -\cancel{2}mc_1 \times \frac{c_1}{\cancel{2}L}$$

$$\boxed{F_1 = -\frac{mc_1^2}{L}} \quad (3)$$

FORCE DUE TO A SINGLE MOLECULE

Assuming a large number of molecules

$$F = F_1 + F_2 + F_3 + \dots + F_N$$

$$|F| = \frac{mc_1^2}{L} + \frac{mc_2^2}{L} + \frac{mc_3^2}{L} + \dots + \frac{mc_N^2}{L}$$

$$|F| = \frac{m}{L}\left(c_1^2 + c_2^2 + c_3^2 + \dots + c_N^2\right)$$

$$\overline{c_x^2} = \frac{(c_1^2 + c_2^2 + c_3^2 + \dots c_N^2)}{N}$$

average of the square of the velocities along the x axis

$$N\overline{c_x^2} = (c_1^2 + c_2^2 + c_3^2 + \dots c_N^2)$$

$$\boxed{F = \frac{Nm\overline{c_x^2}}{L}} \quad (4)$$

Total force on the surface is due to all colliding molecules

$$p = \frac{F}{A} = \frac{F}{L^2}$$

$$p = \frac{Nm\overline{c_x^2}}{L^3}$$

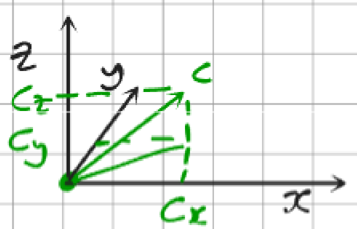
Volume of cube

$$\boxed{p = \frac{Nm\overline{c_x^2}}{V}} \quad (5)$$

Pressure exerted on that surface

So far we have presumed that all particles are moving along the same axis.

Assuming a large number of particles moving in random directions.



$$\overline{c} = \sqrt{c_x^2 + c_y^2 + c_z^2}$$

$$\overline{c^2} = c_x^2 + c_y^2 + c_z^2$$

$$c_x^2 = c_y^2 = c_z^2$$

$$\overline{c^2} = 3c_x^2$$

$$\boxed{c_{rms} = \sqrt{\overline{c^2}}}$$

$$c_{rms}^2 = 3c_x^2$$

$$\boxed{c_x^2 = \frac{1}{3}c_{rms}^2} \quad (6)$$

$$\boxed{p = \frac{1}{3}\frac{Nm\,c_{rms}^2}{V}}$$

Pressure across the container

Nm is the total mass of the gas in the box

$$\boxed{p = \frac{1}{3}\rho\,c_{rms}^2}$$

Comparing gas laws

$$\boxed{pV = nRT} \quad ① \qquad \text{and} \qquad \boxed{pV = \frac{1}{3}Nm\,c_{rms}^2} \quad ②$$

Assuming no intermolecular forces (no change in potential energy)

$$\overline{E_k} = \frac{1}{2}m\,c_{rms}^2 \qquad m\,c_{rms}^2 = 2\overline{E_k}$$

mean kinetic energy

$$pV = \frac{2}{3}N\overline{E_k} \quad ② \qquad nRT = \frac{2}{3}N\overline{E_k} \quad ①$$

$$\overline{E_k} = \frac{3}{2}\frac{Rn}{N}T$$

$$\overline{E_k} = \frac{3}{2}\frac{R}{N_A}T$$

$$\boxed{\overline{E_k} = \frac{3}{2}kT}$$

Mean kinetic energy of the gas.