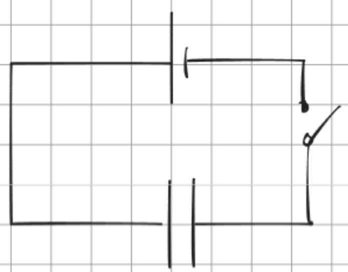
Capacitor Basics

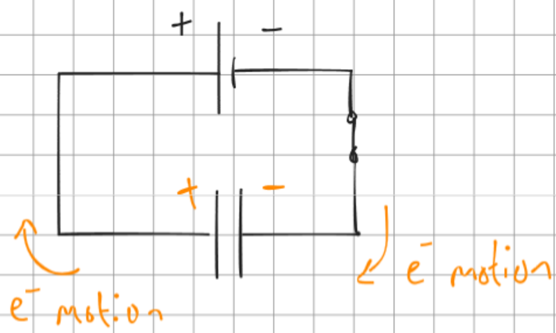
2nd April

Capacitors allow us to STORE CHARGE, which can then be released as needed.

capacitor	battery
energy stored by electric field	stored chemical energy
discharge very quickly, with a decreasing p.d.	discharge slowly with constant p.d.

A capacitor normally consists of two metal plates separated by a dielectric (a type of insulating material - plastics, ceramics, metal oxides).





When the switch is closed electrons are pulled from one of the plates of the capacitor and forced on to the other plate. The side of the capacitor that loses electrons becomes positive, and the side that gains them becomes negative.

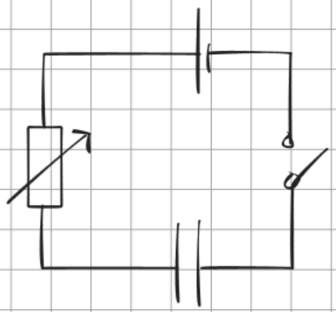
This creates a potential difference across the capacitor (which creates an electric field between them).

As the charge on the plates builds up, so does the p.d. across the plates.

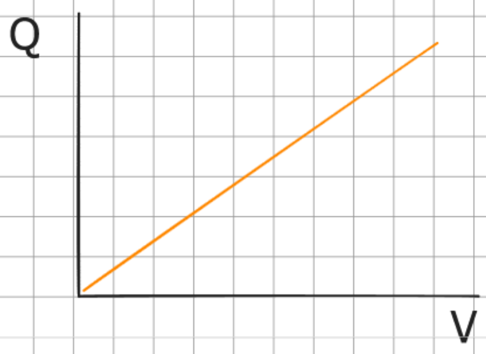
This continues until the p.d. across the plates is equal to the p.d. across the power supply.

Charging with a constant current

Measuring the charge stored on a capacitor is difficult, but we can calculate the stored charge if we keep the current constant.



- When the switch is closed the capacitor charges, but the rate at which charge accumulates would DECREASE over time.
- We can keep this rate constant by reducing the resistance of the variable resistor.
- We can then calculate stored charged: $Q = It$
- If we measure the p.d. across capacitor at regular intervals we can plot Q against V.



$$Q \propto V$$

$$Q = CV$$

Where C is the CAPACITANCE.

$$C = Q/V$$

in Farads, F

A capacitance of 1 F means that 1 C of charge is stored per unit p.d. across the capacitor.

Note: most capacitors have a capacitance in the microfarad range.

When charging a capacitor the battery does WORK.

Energy stored on capacitor = work done when charging

= average charge x V

$$\boxed{E = \tfrac{1}{2} QV}$$

$$\boxed{E = \tfrac{1}{2} CV^2}$$ (as Q = CV)

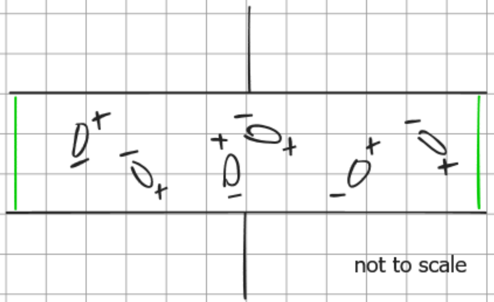
$$\boxed{E = \frac{1}{2}\frac{Q^2}{C}}$$ (as V = Q/C)

Structure of a Capacitor

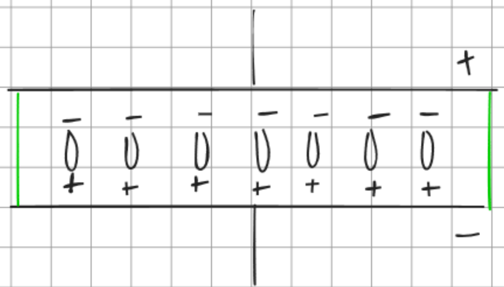
22nd April

We often fill the gap between the plates in a capacitor with a DIELECTRIC.

The dielectrics we choose contain POLAR MOLECULES (one end of the molecule is positive and the other is negative).



When the capacitor is not charged the molecules of the dielectric are randomly orientated.



When the capacitor is charged the polar molecules align with the electric field between the plates.

The combined electric field of the polar molecules acts in the opposite direction to the field between the plates.

This reduces the net electric field in the capacitor (compared to one without a dielectric).

$E = \frac{V}{d}$ hence $V = Ed$. Reducing E reduces V.

By definition $C = \frac{Q}{V}$. With the dielectric we store the SAME CHARGE at a LOWER P.D. Hence the CAPACITANCE has INCREASED.

The PERMITTIVITY of a material is a measure of how easily an electric field can form through it.

The RELATIVE PERMITTIVITY (also known as the DIELECTRIC CONSTANT) is a comparison of the permittivity of the material to the permittivity of free space.

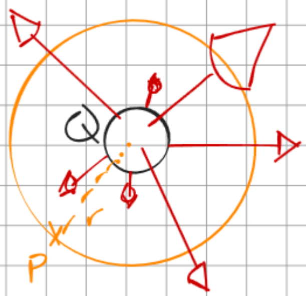
$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

permittivity of the material

dielectric constant (relative permittivity)

permittivity of free space

Area of a capacitor plate



Consider a spherical charge, magnitude Q.

Field strength due to the charge at point P:

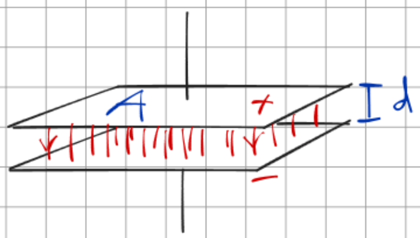
$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$E \times 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$EA = \frac{Q}{\varepsilon_0}$$

'Gauss' law: the net electric flux through any surface is equal to $1/\varepsilon_0$ times the net electric charge enclosed within the surface.'

For a parallel plate capacitor we can consider the charge stored on the surface of the plate, and the surface area of the plate.



$$E = \frac{V}{d}$$

$$EA = \frac{Q}{\varepsilon_0}$$

$$\frac{VA}{d} = \frac{Q}{\varepsilon_0}$$

$$\frac{Q}{V} = \frac{A\varepsilon_0}{d}$$

$$C = \frac{A\varepsilon_0}{d}$$

With a dielectric, we wouldn't the permittivity of free space, we would use the permittivity of the dielectric:

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

surface area of plate

permittivity of free space

$$\boxed{C = \frac{A\varepsilon_0\varepsilon_r}{d}}$$

dielectric constant/relative permittivity

separation of plates

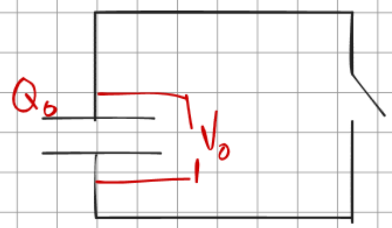
Discharging Capacitors

23rd April

When a capacitor is fully charged it stores charge Q_0

It also has a potential difference across it of V_0

The initial, maximum, current that flows is I_0



When this switch is closed the rate at which charge leaves the plates is at a maximum.

Over time, the potential difference the plates decreases. This causes the rate at which charge leaves the plates to decrease.

$$Q = Q_0 e^{-t/RC}$$

- Q: charge on the plates at time t after discharge has begun
- Q_0: maximum charge stored on plates
- R: Resistance of circuit
- C: capacitance

We define the time constant of a capacitor/resistor combination as

t = RC

When discharging a capacitor after a time equal to the time constant has passed:

$$Q = Q_0 e^{-RC/RC}$$

$$= Q_0 e^{-1}$$

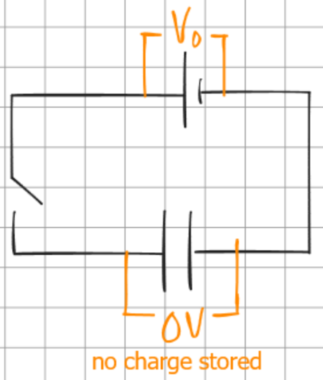
$$Q = 0.368 Q_0$$

The charge on the capacitor has dropped to 36.8% of its original value.

$$Q = Q_0 e^{-t/RC} \xrightarrow{Q = CV} V = V_0 e^{-t/RC} \xrightarrow{V = IR} I = I_0 e^{-t/RC}$$

Charging a Capacitor

25th April



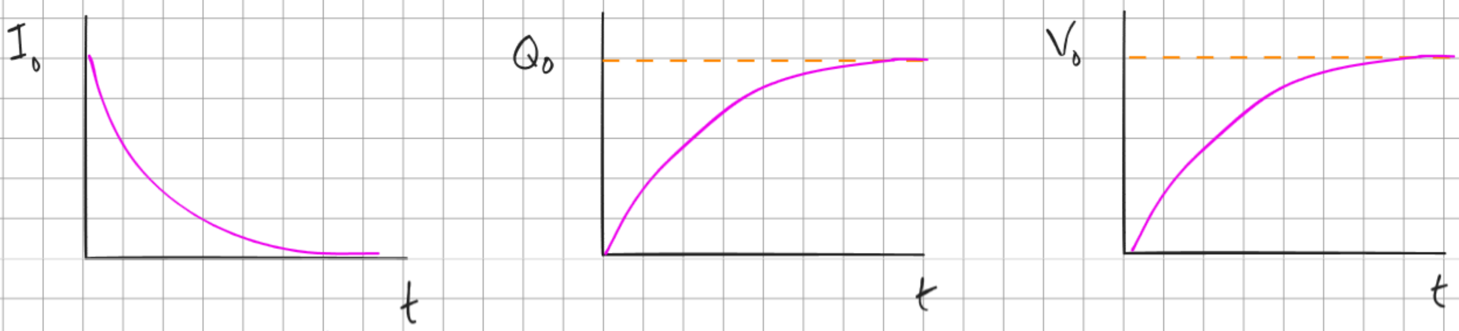
After the switch is closed charge flows and builds up on the plates of the capacitor.

As charge builds up on the capacitor the p.d. across the capacitor increases.

This means that the p.d. between then power supply and the capacitor decreases over time.

This means that, over time, the rate at which charge builds on the plates of the capacitor decreases.

- Charge on capacitor starts at zero and builds to Q_0.
- P.d. across the capacitor starts at zero and builds to V_0
- Current flowing in the circuit starts at I_0 and decreases to zero.



$$I = I_0 e^{-t/RC} \qquad Q = Q_0\left(1 - e^{-t/RC}\right) \qquad V = V_0\left(1 - e^{-t/RC}\right)$$

When t = RC, one time constant has passed.

When charging a capacitor the following is true after one time constant:

$$I = 0.368 I_0$$ (same as when discharging)

$$Q = Q_0(1 - 0.368) = 0.632 Q_0$$

$$V = V_0(1 - 0.368) = 0.632 V_0 .$$