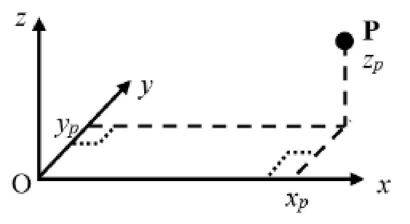
RELATIVITY

To be able to compare measurements it is important to have a common system for recording measurements. This system is known as a FRAME OF REFERENCE.

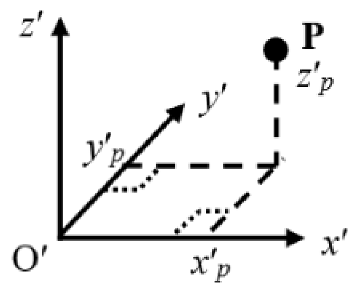
Frames of reference

To measure an object's position requires a 3 coordinate system.
To do this requires a FRAME OF REFERENCE, with three coordinates, often referred to a x,y and z, corresponding to three different axes.

The position of an object P measured by an observer O would be at position ($x_p,\ y_p,\ z_p$)



If P were an EVENT rather than an object, or if P were an OBJECT in MOTION then we may also require a time to be recorded, t_p.



A different observer O' examining the same object will have a different FRAME OF REFERENCE, with a coordinate system of ($x'_p,\ y'_p,\ z'_p$)
And a particular time of t'_p

Absolute space and time

According to Newton and Galileo, provided that O and O' synchronise their clocks, then $t_p = t'_p$ and they will record the same time for events.
This leads to the concept of ABSOLUTE TIME.

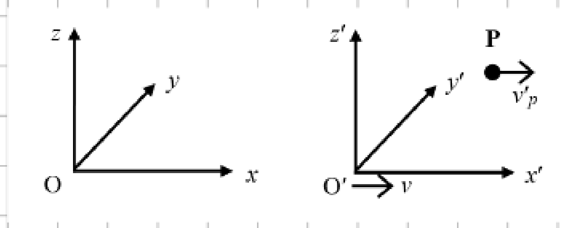
Different observers may be making observations from different points, but they should agree on the lengths or distances between objects.
This is known as ABSOLUTE SPACE.

<u>Relative velocities</u>

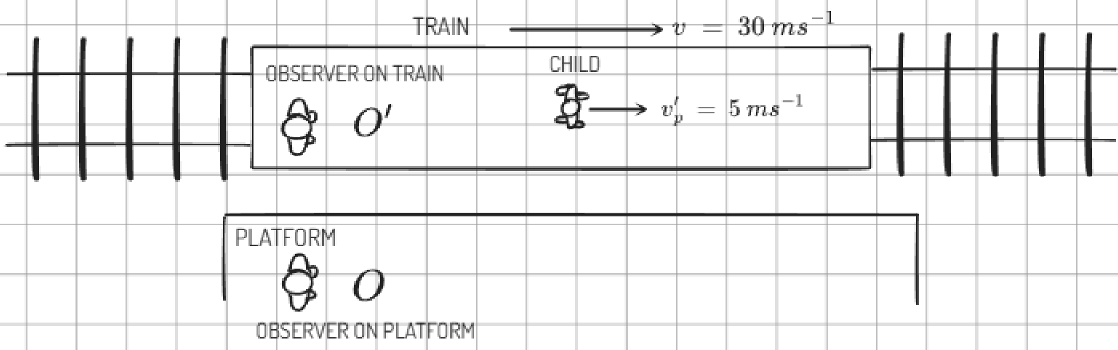
If the two observers are moving relative to each other, so that in the frame of reference of observer O, observer O' is moving at a velocity of v.

Observer O' measure the velocity of P to be v'_p.

According to relativity:

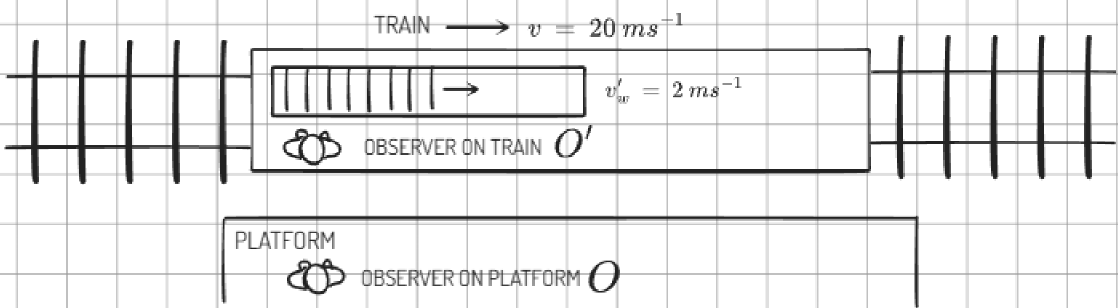


$$v_p = v'_p + v$$



From the frame of reference of O' the child is moving at $5\,ms^{-1}$

From the frame of reference of O the child is moving at $30 + 5 = 35\,ms^{-1}$



From the frame of reference of O the speed of the water waves would be given by:

$$v_w = v'_w + v$$

The same argument could be applied to sound waves, giving the result:

$$v_s = v'_s + v$$

And again to light, giving the result:

$$c = c' + v$$

NEWTONIAN RELATIVE REQUIRES A NON-CONSTANT SPEED OF LIGHT!

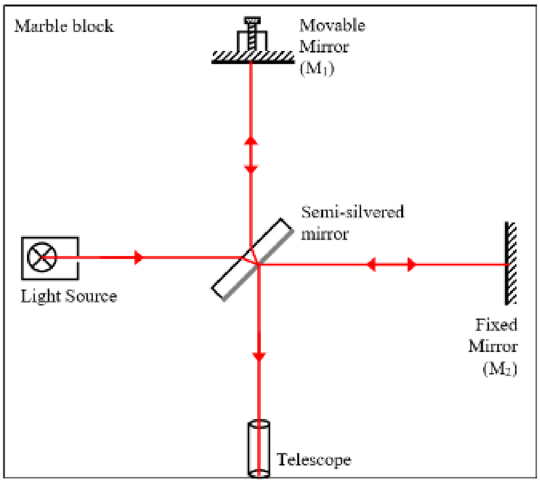
THIS THEORY DOES NOT WORK!

THE ETHER

It was believed that light, just as all other waves, needs a medium through which to travel.
This medium was known as the LUMINIFEROUS ETHER.
The ETHER needed to fill empty space to allow light from distant stars to reach us. The Ether formed the basis of absolute space, all lengths and times can be measured against it.

So that the presence of the Ether did not effect the motion of the planets it must be MASSLESS, and be WITHOUT VISCOSITY.
To support high frequency vibrations it must also be RIGID and INCOMPRESSIBLE.

The Michelson-Morley Interferometer



The interferometer light is directed towards a semi-silvered mirror.
At the mirror, half the intensity is reflected to M1, and half transmitted to M2.
Some of each beam passes back through the semi-silvered mirror to the telescope.
The two beams of light reflected by M1 and M2 combine and interfere.

By moving M1, the path difference between the two waves can be adjusted, changing the pattern of the fringes.

The interferometer was mounted on a marble block to prevent disturbance by vibration over long periods.
This was mounted on liquid mercury to allow the apparatus to rotate.

The Michelson-Morley experiment

If the theory of the Ether were correct, rotating the apparatus should change the speed of light down the different paths, and result in the light travelling different path lengths.
This would change the path difference between the two beams and lead to a measurable change in the interference pattern.

NO SIGNIFICANT OBSERVATIONS WERE MADE.

The aim of the experiment was to measure the speed of Earth through the Ether. This is one of the most important failed experiments in history, as it instead proved the INVARIANCE of the speed of light and opened the door to Einstein's theory of special relativity.

SPECIAL RELATIVITY - TIME AND SPACE

Einstein's Postulates

One important concept is the idea of an INERTIAL FRAME OF REFERENCE. This is a frame of reference in which Newton's Second Law applies.

If one frame of reference is an inertial frame, then any other frame of reference moving at a CONSTANT VELOCITY relative to the frame is also inertial.

The first postulate:

Physical laws have the same form in all INERTIAL frames of reference.

This means no inertial frame can be singled out as 'at rest'
Absolute space is a meaningless concept. ALL MOTION is relative motion.

The second postulate:

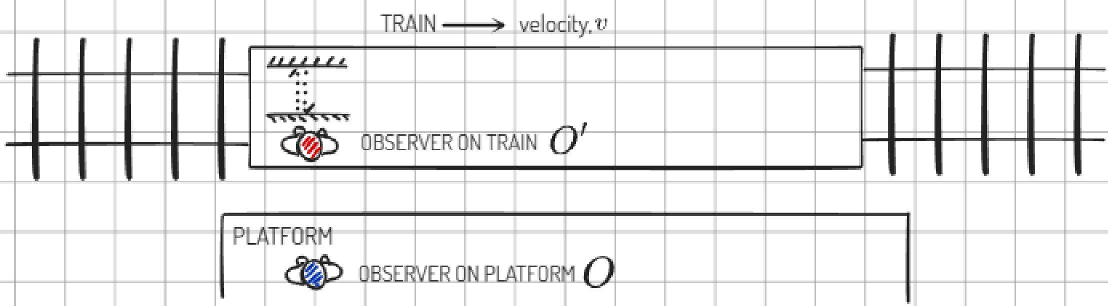
The speed of light in free space is the same when measured in any frame of reference.

TIME DILATION

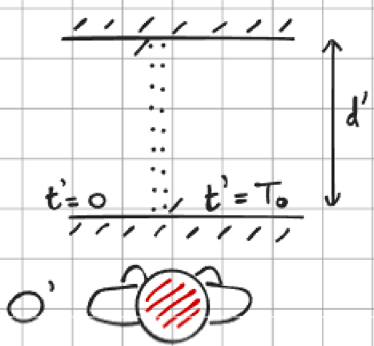
The time interval measured in the frame of reference which is AT REST relative to two events is known as the PROPER TIME T_0
(The two events happened in the same place in that frame of reference)

Thought experiment - a light clock on a train

A light clock is made by bouncing a pulse of photons between two mirrors. The motion of the photons corresponds to the oscillation of a pendulum in a traditional clock.



On the train



The observer on the train has his own reference frame.
He measures the time taken for the pulse of light to travel from one mirror, to the other and back again.

The FIRST EVENT is the pulse leaving the first mirror,
the SECOND EVENT is the light returning to the first mirror.

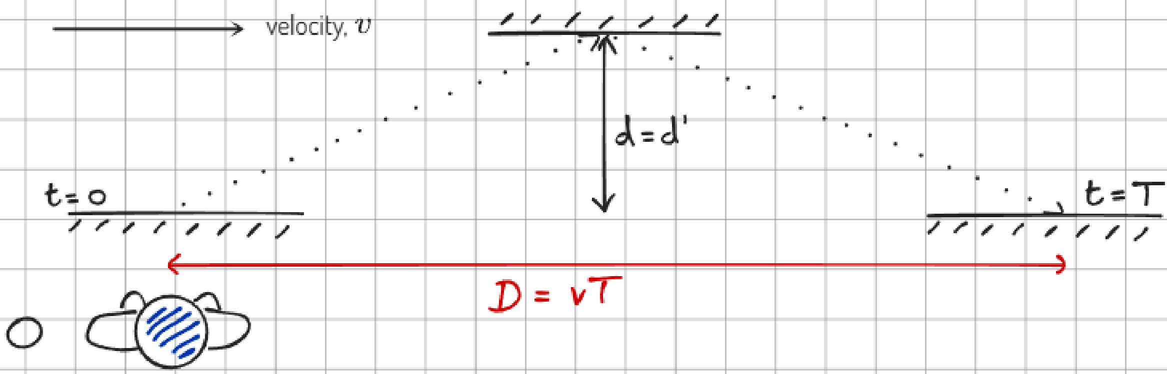
This observer is AT REST relative to these events, so measures PROPER TIME.

$$d' = \frac{cT_0}{2} \quad (1)$$

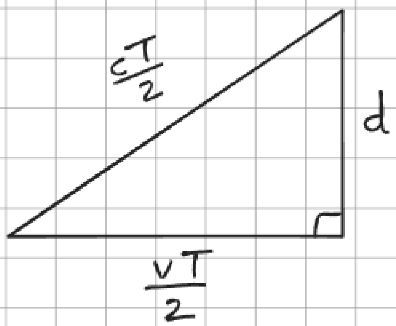
On the platform

The observer on the platform has his own reference frame. The train appears to move past at a velocity, v.

Therefore the first and second events DO NOT happen in the same place in his frame of reference.



The time interval observer is NOT proper time. This observer records T.
The light travels a distance of $\frac{cT}{2}$ as the mirror travels a distance of $\frac{D}{2}$



Using Pythagoras Theorem:

$$\frac{c^2T^2}{4} = d^2 + \frac{v^2T^2}{4}$$

Using ① $\quad d = d' = \frac{cT_0}{2}$

$$\frac{c^2T^2}{\not{4}} = \frac{c^2T_0^2}{\not{4}} + \frac{v^2T^2}{\not{4}}$$

$$c^2T^2 = c^2T_0^2 + v^2T^2$$

$$(c^2 - v^2)T^2 = c^2T_0^2$$

$$\left(1 - \frac{v^2}{c^2}\right)T^2 = T_0^2$$

$$T^2 = \frac{T_0^2}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$\boxed{T = \frac{T_0}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}}$$

The time measured by the observer on the platform will be LONGER than the PROPER TIME. According to relativity, TIME ITSELF will be slower for this observer.

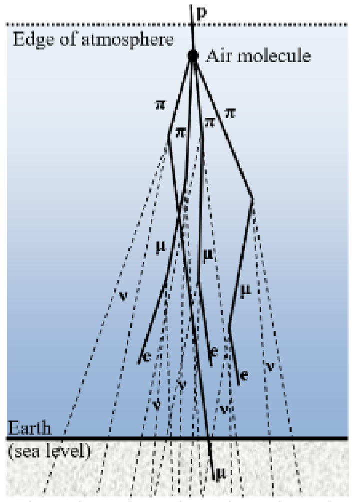
EVIDENCE FOR TIME DILATION

TIME DILATION

The time interval measured in the frame of reference which is AT REST relative to two events is known as the PROPER TIME T_0
(The two events happened in the same place in that frame of reference)

$$T = \frac{T_0}{\left(1-\frac{v^2}{c^2}\right)^{1/2}}$$

Muons are produced by cosmic rays about 15 km above sea level. They travel down towards at almost the speed of light.



Muons are unstable and decay according to:

$$N = N_0 e^{-\lambda t} \quad \text{with} \quad \lambda = \frac{\ln 2}{t_{1/2}}$$

(To use the equation above the half-life and time must be measured in the SAME REFERENCE FRAME)

When the measured in the MUON'S REST FRAME, the muons decay with a half-life of $1.523 \times 10^{-6}\ s$

In the frame of reference from an observer on earth, it takes $5.01 \times 10^{-5} s$ for the muons to reach sea level.

$$v = \frac{L}{T} = \frac{5\times10^{3}}{5.01\times10^{-5}} = \boxed{2.994\times10^{8}\ ms^{-1}}$$

$$\frac{5.01\times10^{-5}}{1.523\times10^{-6}} = 33 \text{ half lives} \qquad \frac{1}{2^{33}}\times100 = \boxed{1.164\times10^{-8}\%}$$

The observer at sea level detects 25% of the original muons.

The half-life and travel time are in DIFFERENT REFERENCE FRAMES.
The EVENT is the muon decay, so the muon experiences PROPER TIME.

$$T_{1/2}\text{ (observer)} = \frac{T_{1/2}\text{ (proper)}}{(1-v^2/c^2)^{1/2}}$$

$$= \frac{1.523\times10^{-6}}{(1-0.996)^{1/2}}$$

$$= \frac{1.523\times10^{-6}}{(1-0.996)^{1/2}}$$

$$= \frac{1.523\times10^{-6}}{0.0631} = \boxed{2.4\times10^{-5}s \text{ for observer}} \rightarrow 2.08 \text{ half lives}$$

OR

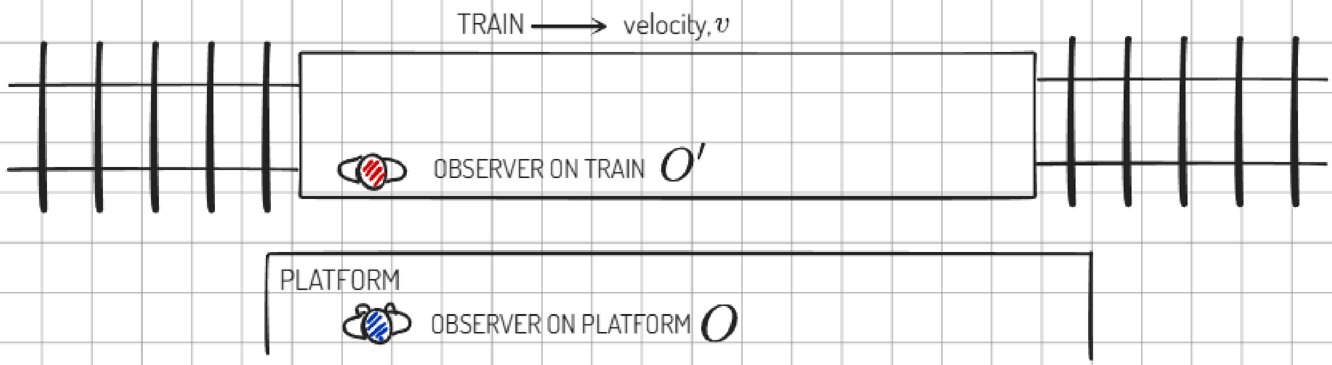
$$5.01\times10^{-5} = \frac{T\text{ (proper)}}{(1-v^2/c^2)^{1/2}}$$

$$T\text{ (proper)} = 5.01\times10^{-5}\times0.0631$$

$$= \boxed{3.17\times10^{-6}s \text{ for muons}} \rightarrow 2.08 \text{ half lives}$$

If either object is forced to ACCELERATE, then the frames of reference become NON-INTERTAIL and special relativity cannot be applied.

LENGTH CONTRACTION



Each of these observers is going to measure the length of the platform.

Observer on the train

For the observer on the train O' records the time it takes for them to cover the length of the platform. From their frame of reference, the platform appears to be moving at speed v.

They start at one end of the platform, and when they arrive at the other end of the platform, the two ends appear to be in the same place.

This observer records PROPER TIME.

The length of the platform appears to be:

$$L' = vT_0$$

Observer on the platform

For the observer on the platform records a time of T for the train to traverse the platform, at a speed of v.

The length of the platform can be calculated as vT.

This observer is AT REST relative to the platform. So they measure the PROPER LENGTH of the platform.

$$T = \frac{T_0}{(1-v^2/c^2)^{1/2}}$$

$$\boxed{T = T_0(1-v^2/c^2)^{-1/2}}$$

$$L_0 = vT$$

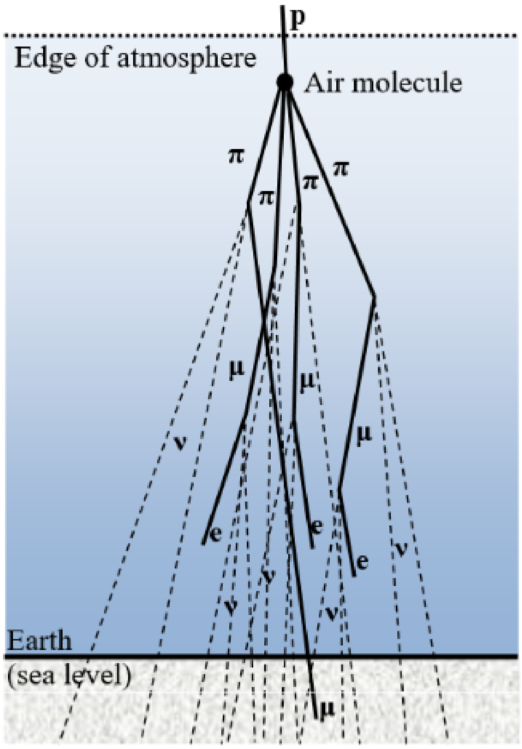
$$L = vT_0$$

$$vT = \frac{vT_0}{(1-v^2/c^2)^{1/2}}$$

$$L_0 = \frac{L}{(1-v^2/c^2)^{1/2}}$$

$$\boxed{L = L_0(1-v^2/c^2)^{1/2}}$$

So the platform appears SHORTER to the observer on the train.



$$L = 15\times10^3(1-0.998^2)^{1/2}$$
$$= 948\,m$$

$$\frac{948}{3.17\times10^{-6}} = 2.99\times10^8\ ms^{-1}$$

PROPER travel time from muons frame of reference

Relativistic Mass

Rest mass (m_0): The mass of an object as measured by an observer AT REST relative to the object.

Relativistic mass (m) : The mass of an object as measured by an observer MOVING relative to the object.

It is found that:

$$m = \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

i.e. the mass of an object increases with velocity

Relativistic Momentum

$p = mv$ is still true for objects with relativistic velocities.
But mass is relativistic, so:

$$p = \frac{m_0 v}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

Relativistic Energy

Einstein deduced that mass was a form of energy store, such that:

$$E = mc^2$$

where m is the relativistic mass, so:

$$E = \frac{m_0 c^2}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

In the limit v = c, the particle would have INFINITE ENERGY. So, it is IMPOSSIBLE to accelerate a particle with non-zero mass to the speed of light. The closer an object gets to the speed of light, the more the mass increases, and the lower the rate of increase in velocity.

For an object AT REST, v = 0, so $E_0 = m_0 c^2$ gives the REST MASS ENERGY of the object.

$$mc^2 = \frac{m_0c^2}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

$$m^2c^4 = \frac{m_0^2c^4}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$m^2c^4\left(1 - \frac{v^2}{c^2}\right) = m_0^2c^4$$

$$m^2c^4 - m^2c^2v^2 = m_0^2c^4$$

$$E^2 - p^2c^2 = m_0^2c^4$$

$$E^2 = m_0^2c^4 + p^2c^2$$

If REST MASS is zero:

$$E^2 = p^2c^2$$

Using de Broglie's equation:

$$E^2 = \frac{h}{\lambda^2}^2 c^2$$

Using Planck's equation of quanta:

$$h^2f^2 = \frac{h}{\lambda^2}^2 c^2$$

Make c the subject:

$$c = f\lambda$$

Relativistic Kinetic Energy

When an object is accelerated, it gains kinetic energy, K.
The total energy of the particle is equal to the kinetic energy plus the rest mass energy.

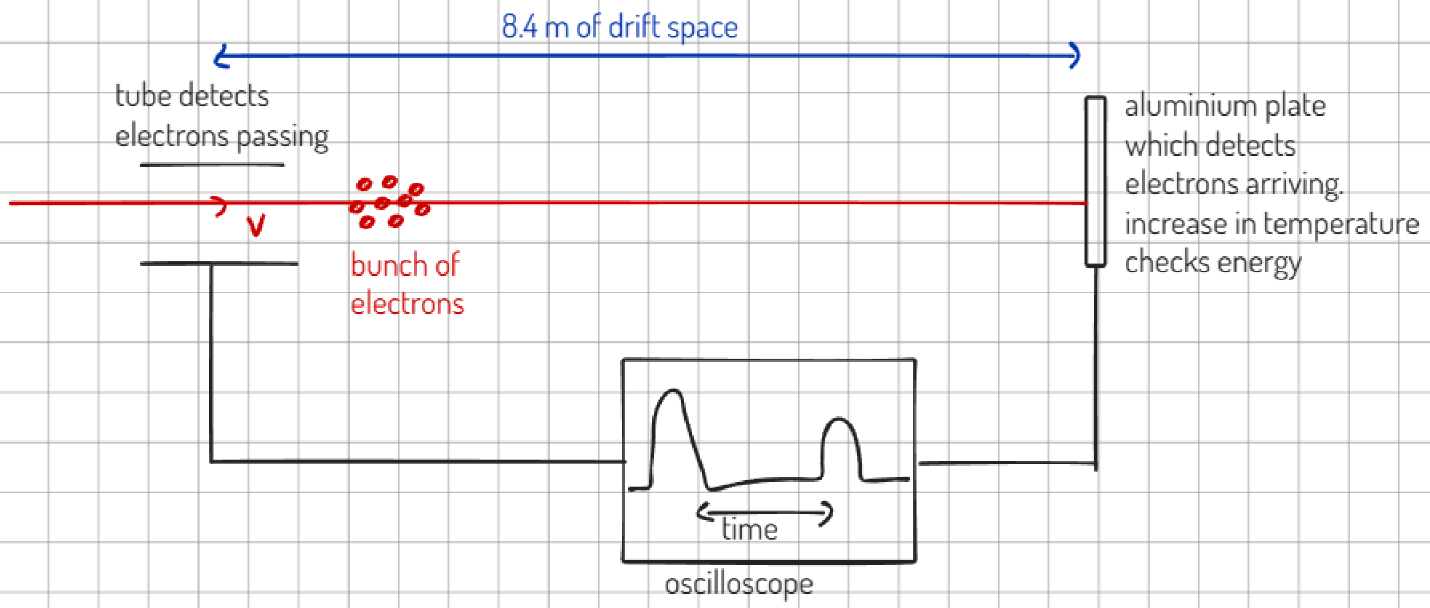
$$E = E_0 + K$$

The relativistic kinetic energy is given by:

$$K = E - E_0$$

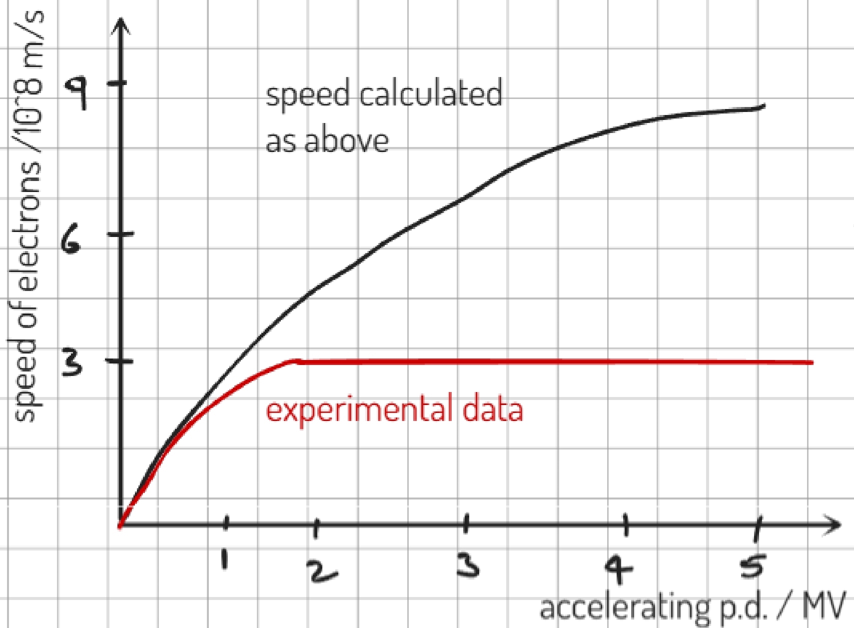
$$K = mc^2 - m_0c^2$$

Bertozzi's experiment – The Ultimate Speed



The accelerated electrons will have a velocity which depends on the accelerating potential difference.

$$\frac{1}{2}mv^2 = qV$$



When considering high energy accelerato
the speed of a particle is limited to c
but Kinetic Energy and Momentum can
theoretically be increased WITHOUT LIMIT
As v approaches c, K tends to infinity.

For velocities greater than 0.2c we cannot apply classical equations for motion.