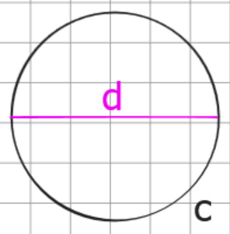
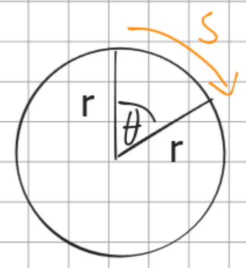
Circular Motion Basics

19th May



$\pi = \frac{c}{d}$



One radian is the angle subtended at the centre of a circle by an arc length equal to the radius of the circle.

(in radians) $\theta = \frac{s}{r}$ (when s = r, θ = 1)

In one revolution: $s = C$ and $c = \pi d$

$c = 2\pi r$ (as d = 2r)

$\theta = \frac{2\pi r}{r}$

$\theta = 2\pi$ - so a circle has a total angle of 2π radians.

When converting to and from degrees: $2\pi = 360^\circ$

$\pi = 180^\circ$

So to convert from radians to degrees: $\frac{\theta}{\pi} \times 180$

degrees to radians: $\frac{\theta}{180} \times \pi$

Time period: time taken to complete one full revolution.

Frequency: the number of full revolutions completed per second

$\boxed{T = \frac{1}{f}}$

Linear speed, v, can be found from:

$v = \frac{\text{total distance travelled}}{\text{time taken}}$ $v = \frac{2\pi r}{T}$ (a full revolution)

Angular speed, ω, is the angle subtended per second.

$\omega = \frac{\theta}{t}$ — in radians, in seconds; rad s^{-1}

Learn as a definition

In one revolution: $\boxed{\omega = \frac{2\pi}{T}}$ or $\boxed{\omega = 2\pi f}$

Given

Note: these are quite useful equations to learn

We know that: $v = \frac{2\pi r}{T}$

$$\boxed{\begin{gathered} v = \omega r \\ \omega = \frac{v}{r} \end{gathered}}$$

Given

Acceleration in Circular Motion

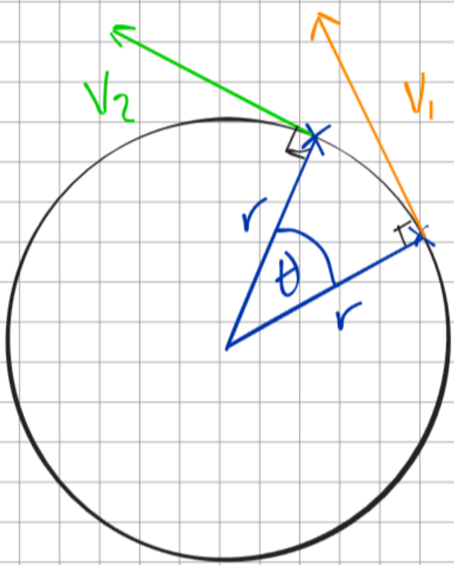
20th May

Velocity is a vector, it has a magnitude and a direction.

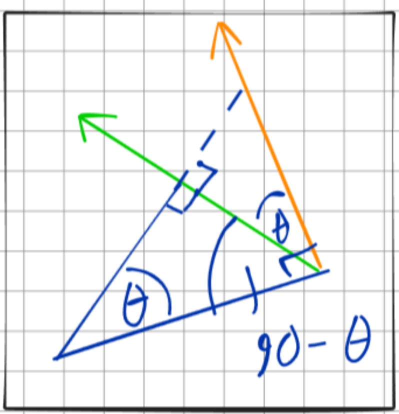
If an object moves in a circular path then its direction is constantly changing, hence its velocity is constantly changing.

This means, by definition, the object is constantly accelerating (as acceleration is the rate of change of velocity).

$a = \frac{\Delta v}{t}$ by definition

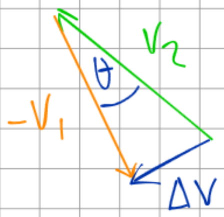


Velocity vectors for an object at two different positions on a circular path with radius r.



This is to show that the angle between the two velocity vectors is the same as the angle at the centre of the circle.

Change in velocity = $\Delta v = v_2 - v_1 = v_2 + (-v_1)$



This is the arc of a circle with a radius equal to the magnitude of the velocity vectors.

At small angles: $s \approx \Delta v$

and we know, if the angle is in radians that:

$$\theta = \frac{s}{v}$$

$$\omega = \frac{\theta}{t}$$

$$v = \omega r$$

$$s = v\theta$$

$$s = v\omega t$$

$$s = \omega r \omega t$$

$$s = \omega^2 r t$$

$(s \approx \Delta v)$

$$\Delta v = \omega^2 r t$$

$$a = \frac{\Delta v}{t}$$

$$\boxed{a = \omega^2 r}$$

$$a = \left(\frac{v}{r}\right)^2 r$$

$$\boxed{a = \frac{v^2}{r}}$$

These tell us the magnitude of the acceleration of an object moving in a circular path

Definition of angular speed: $\omega = \frac{\theta}{t}$

Other useful equations with angular speed: $\omega = 2\pi f$

$$\omega = \frac{2\pi}{T}$$

T is period: time for one revolution

f is frequency: revolutions per second

$$f = \frac{1}{T}$$

Linear speed = $\frac{\text{distance travelled}}{\text{time}}$: $v = \frac{2\pi r}{T}$

Link between linear and angular speed: $v = \omega r$

Acceleration in circular motion (centripetal acceleration)

$$a = \omega^2 r$$

$$a = \frac{v^2}{r}$$

Resultant Forces in Circular Motion

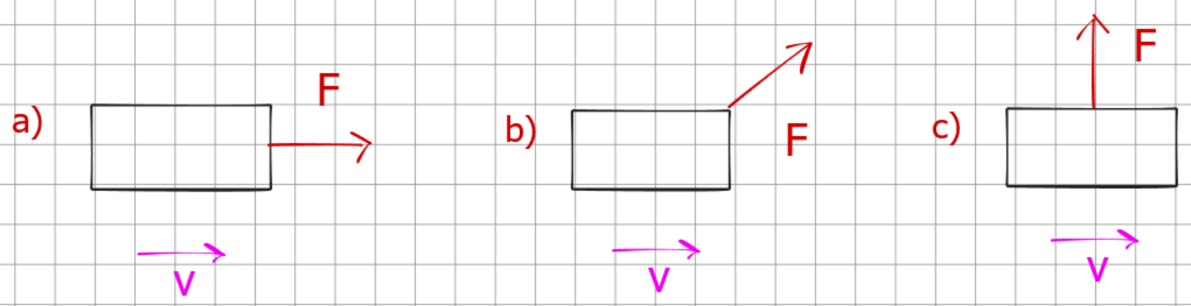
2nd June

According to N1L a RESULTANT FORCE is required to cause an object to accelerate.

There MUST be a resultant force that acts on ALL objects that are moving in a circle.

For an object moving in a circular path at a constant speed the resultant ALWAYS acts RADIALLY INWARDS.

Consider these different scenarios:



v is the velocity of the object, and F represents the RESULTANT force on the object.

In a) the resultant force is parallel, and in the same direction as, the objects velocity. The magnitude of v will increase, but its direction will not change.

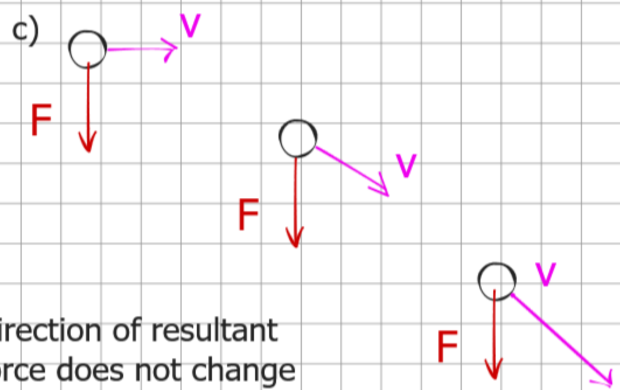
The kinetic energy of the object increases.

In b) there is a component of the resultant force parallel to v. The magnitude of v increases, as does the object's kinetic energy.

There is also a component of F perpendicular to v; this will change the direction of v and the object will start to follow a curved path.

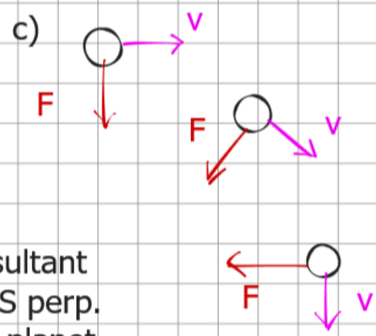
In c) F is perpendicular to v; the force can only change the direction of the object and not its speed.

Projectile motion



Direction of resultant force does not change e.g. object launched horizontally in a gravitational field

Circular motion



Direction of resultant force is ALWAYS perp. to velocity e.g. planet in orbit around a star

In circular motion we call the resultant force the CENTRIPETAL FORCE (centri 'centre' petal 'seeking'), as it always acts towards the CENTRE OF THE CIRCULAR PATH.

From N2L: Resultant force = mass x acceleration

In circ. motion Centripetal force = mass x acceleration

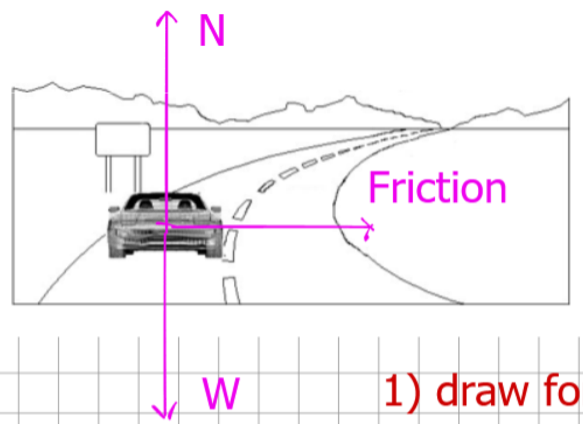
$$F = ma$$

where $a = \frac{v^2}{r}$ or $a = \omega^2 r$

Hence $F = \frac{mv^2}{r}$ or $F = m\omega^2 r$

F here is the resultant force causing circular motion i.e. the centripetal force

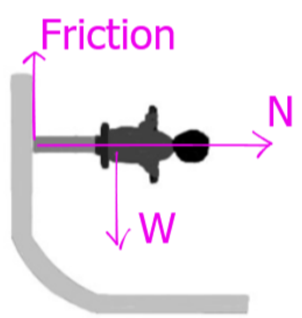
Car along a level curve



- We don't add 'centripetal force' as a separate arrow as it is just the resultant of the forces that act on the obect.
- If the object is moving in a circle we know there MUST be a resultant and it MUST be radially inwards.

1) draw force/free body diagram
2) resolve forces vertically and horizontally
3) write an expression for resultant (centripetal) force

'Wall of Death' Motorcycle Stunt

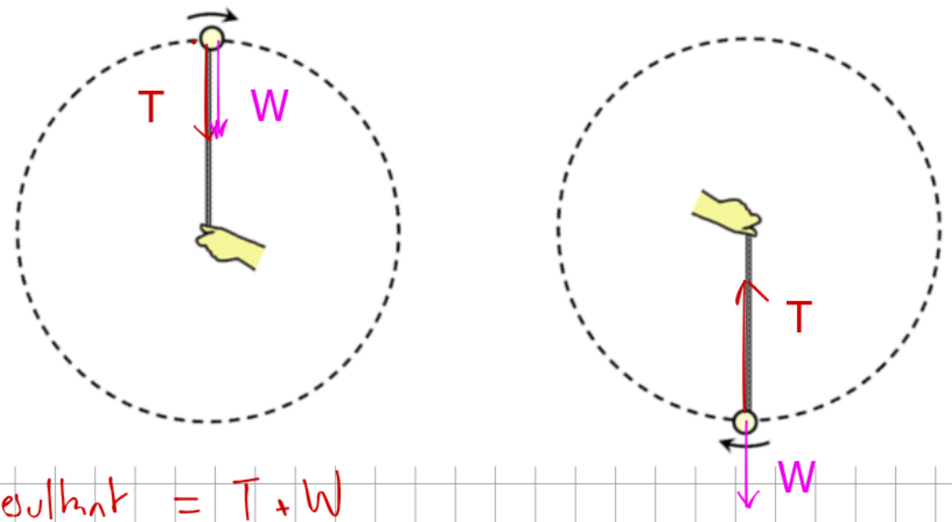


Vert; Friction = W (in eq.).

Horizontally; Resultant (centripetal) = N

$$\frac{mv^2}{r} = N$$

Mass ‘m’ moving in vertical circle radius ‘r’ at speed ‘v’



Resultant = T + W

$$\frac{mv^2}{r} = T + mg$$

$$T = \frac{mv^2}{r} - mg$$

Minimum tension

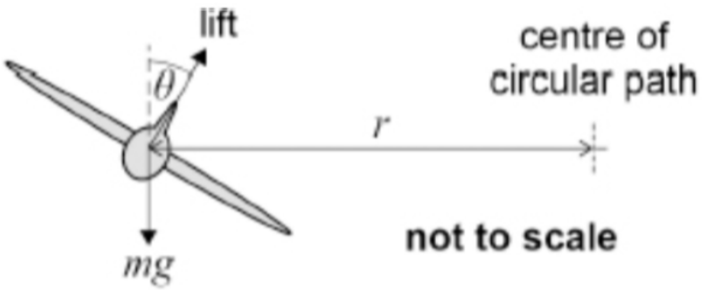
Resultant = T − W

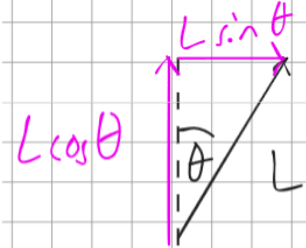
$$\frac{mv^2}{r} = T - mg$$

$$T = \frac{mv^2}{r} + mg$$

Maximum tension

Banking Aircraft





Vert; $L\cos\theta = mg$

$$\boxed{L = \frac{mg}{\cos\theta}}$$

Horizontally; Resultant $= L\sin\theta$

$$\frac{mv^2}{r} = L\sin\theta \quad , \quad \boxed{L = \frac{mv^2}{r\sin\theta}}$$

We can equate these two expressions for L to help eliminate L and help us find an expression for v:

$$\frac{\cancel{m}g}{\cos\theta} = \frac{\cancel{m}v^2}{r\sin\theta}$$

$$v^2 = \frac{rg\sin\theta}{\cos\theta} \qquad v^2 = rg\tan\theta$$

$$v = \sqrt{rg\tan\theta}$$